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Turbulence in Fluid Dynamics: Theoretical Models and Scaling Laws

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Abstract-

Turbulence, a persistent enigma in classical physics, characterizes chaotic and nonlinear fluid motion across a wide range of natural and engineered systems. Despite significant theoretical and computational advances, a unified framework for understanding turbulence remains elusive. This paper presents a conceptual analysis of turbulence in fluid dynamics, focusing on major theoretical models and scaling laws that have shaped the discipline. From Kolmogorov's inertial-range hypotheses to modern simulations such as DNS and LES, the study outlines how dimensional analysis, self-similarity, and multifractal models contribute to the scaling behavior of turbulent flows. It further explores the energy cascade mechanism and the implications of turbulence in real-world contexts such as meteorology, aerospace engineering, and astrophysics. By comparing various turbulence modeling strategies and highlighting their strengths and limitations, the paper offers insight into ongoing challenges and future directions in turbulence research. This work aims to provide a non-mathematical yet comprehensive understanding of turbulence for physicists and interdisciplinary researchers.

Keywords: Turbulence, Fluid Dynamics, Scaling Laws, Energy Cascade, Dimensional Analysis, Self-Similarity, Nonlinear Systems, Multifractal Models.

Introduction

Turbulence represents one of the most challenging problems in classical physics. Its general quantitative description remains incomplete, despite contemporary methods encompassing laboratory experiments , numerical simulations , and field measurements . For these reasons, the statistical study of simplified theoretical models continues to be notable; these models often represent reduced physical systems displaying fundamental features of turbulence. This line of research applies interestingly to models inspired by Navier–Stokes (NS) equations (Bratanov, 2015) as well as to more phenomenological approaches . This paper considers a generalized three-dimensional turbulent model able to provide new insights on turbulent dynamics, concentration on the particularly interesting and complex interplay between wave and turbulent dynamics.

The consideration of turbulent relaxation, in a context where two dynamical processes coexist and compete, represents an important open problem. Its occurrence is frequent in physics, as evidenced, for instance, in geophysical dynamics where turbulent and wave phenomena coexists . The incompressible Navier–Stokes equations constitute the standard framework for the study of three-dimensional turbulence. Their turbulent features are characterized by the presence, in a steady state, of an energy flux moving over a broad range of scales; this flux ensures the progressive flow of energy from the injection scale towards the viscous scale where the dissipative process acts strongly. The well-known Kolmogorov theory provides the classical description of this transfer mechanism. The resulting energy spectrum at small wave numbers follows a K”5/3 behavior; at large wave numbers, the response depends on the behavior of the viscous term, which can be either standard viscosity (resulting in an exponential decay of the energy spectrum), or hyperviscosity, as often used in simulations .

Historical Background

The study of turbulence in fluid dynamics has a rich historical background that spans more than a century. The first evidences of turbulence formation by a transition from laminar flow around solid objects were reported by Reynolds, who described them by introducing the Reynolds Number to discriminate them from laminar flows. The Kolmogorov theory offers a framework for describing the phenomenology of turbulence based on the inertial mechanisms that determine cumulative and segmental fluctuations in the energy cascade process for turbulent velocity fields. The theory is based on the assumption that, at very high Reynolds numbers, the statistics of the small-scale motion in the range of scales, where viscosity effects are negligible, are universally and uniquely determined by the kinematic viscosity and the mean rate of dissipation of turbulence kinetic energy per unit mass (A. Khossousi, 1987).

Key Theories of Turbulence

Energy spectrum of the fluctuations is an important tool in turbulence studies. Kolmogorov predicted that in the inertial range, the spectrum of the velocity field is proportional to k”5/3 (in the inertial range). Subsequently, other spectral exponents were derived for other fields, dimensions, and systems. It is also possible to derive other spectra from various geometry symmetries.

Experimental and numerical observations of passive-scalar spectra in 3-dimensional turbulence showed a k”5/3 scaling of temperature that matches with Kolmogorov’s spectrum (1941 model). Other experiments in the inertial–convective range, for instance, by Eckelmann and Shin and Hanratty, also supported a k”5/3 dependence on the wave number.

However, the Batchelor–Howells–Townsend model predicted a dependence with slope “1 in the inertial subrange for the spectral density of a passive scalar. A scalar with very small molecular diffusivity acts as a passive tracer and has been investigated in numerous experimental and numerical studies. The departure of empirical results from the Batchelor spectrum could be attributed to limitations of the model, sampling effects, or assumptions on the source distribution.

Kolmogorov’s Theory

Kolmogorov’s phenomenological theory of turbulence forms the backbone for the understanding of turbulent flows at high Reynolds numbers. The theory is applicable to fluids in turbulent regimes when the statistics of the velocity field can be reasonably approximated as homogeneous and isotropic. The concepts were elucidated in his seminal works . The entire flow is conceived as an amalgamation of eddies of various sizes, with energy cascading from larger to smaller scales until it is ultimately dissipated at the Kolmogorov length scale . The core of the theory is the construction of a self-similar, anisotropic velocity field characterized by a universal scaling exponent that governs the relationship between the Reynolds number and the moments of the velocity increments.

Reynolds’ Contributions

Straightforward scaling arguments based on the Navier-Stokes equations give an estimate of the Reynolds number at which a flow loses regularity. Independently, the same estimate appears if one assumes that the transition from laminar to turbulent flow is a dynamical system bifurcation. In a turbulent regime, the Navier-Stokes equations, unlike a smooth laminar flow, do not generate a relevant extra constant to build a scaling theory. This holds for the energy dissipation and for stress on the surface of a projectile moving in the flow because, when the viscosity is neglected, the equations become the inviscid Euler equations that, in the stationary limit, cannot generate any dynamical scale (Pomeau & Le Berre, 2019). Therefore, dissipation in the limit of large Reynolds numbers should be proportional to the kinetic energy u2 of large-scale fluctuations, multiplied by a characteristic frequency ù independent of the Reynolds number Re, which fixes a velocity scale and an effective (turbulent) viscosity, and determines scale ratio for velocity fluctuations. In contrast, the commonly used assumption of a constant rate of energy dissipation per unit volume and density leads to the turbulent viscosity proportional to u3/L, with L the energy-containing scale, and to a Reynolds-number dependent scale ratio. The observed dependence of the transition Reynolds number on the relative amplitude of a finite perturbation finds a formal explanation in the bifurcation framework, which predicts a coefficient of proportionality very close to the measured value. Finally, the case of a purely temporal instability shows how a supercritical bifurcation can reconstruct the rise and fall of amplitude when the Reynolds number is swept through the transition value.

Energy Cascade in Turbulence

Turbulent flows encompass velocity fluctuations across a broad spectrum of scales, from the largest scales, where energy input occurs, down to the smallest scales, where viscous effects dissipate energy into heat. Similar scale-dependent phenomena are found in diverse contexts such as geophysical currents, plasma dynamics, solar corona activity, and atmospheric pollutant dispersion (Ran, 2010). In fully developed turbulent flows, the Reynolds number1 dimensionless quantity representing the ratio of inertial to viscous forces1 must be extremely large. This conditions the wide separation between the energy injection scale and the dissipative scales, enabling the possibility of an inertial interval over which energy flows from large to small scale fluctuations without interruption. The energy cascade remains at the core of even the most advanced turbulence analyses (Düring et al., 2019) and can be viewed as a nonlinear transfer of energy through the scales of the flow: an energy flux directed towards smaller and smaller scales carries the energy injected by the external forcing to the high-frequency modes where it is dissipated. As originally envisaged by Richardson, the transfer process constitutes a nontrivial spatio-temporal interplay between the large- and small-scale vortex structures (Reynoso et al., 2023) , a transfer that ultimately finds its origin in the scale-locality of the nonlinear interactions characterizing the NS equation.

Scaling Laws in Turbulence

Brilliant arguments by Kolmogorov based on scale invariance of the inviscid Navier Stokes equations lead to predictions about the statistics of velocity increments. This enables an understanding of the associated phenomenon: scale-dependent anomalous scaling exponents and the associated breakdown of self-similarity in increments at inertial-range scales. Scale invariance is broken in either of two ways: by the set-up or boundary conditions of the experiment or, as in turbulence, spontaneously by the dynamics of the system. Although Kolmogorov believed the conditions of the experiment would be important by analogy with critical phenomena, the scale interaction in the non-linear term of the Navier Stokes equation leads to anomalous exponents that depend neither on the forcing nor the boundary conditions. Following Kolmogorov's original reasoning, the anomalous exponents can be deduced solely from the symmetry of the equations of motion of the fluid and the existence of a pivotal point in the distribution of the velocity increment at a scale in the inertial range.

Dimensional Analysis

The governing equations that describe fluid flow are those derived by Stokes: the Navier-Stokes equation together with the incompressibility condition, where the velocity field and pressure gradient appear symmetrically. These equations hold regardless of the Reynolds number (Re) and thus apply equally to laminar and turbulent flows (A. Khossousi, 1987) (R. Sreenivasan & Yakhot, 2021). In the limit of very small Re, scaling follows from the viscous term of the Navier-Stokes equation together with the incompressibility condition. Laminar flow then justifies the further use of simple linear propagators. Although flow is not laminar in every situation, the viscous term always dominates for wave number q far enough beyond the dissipative wave number q_d. For q k" q_d, the scaling of velocity derives, as in the laminar flow case, from the viscous term and incompressibility condition without any additional hypothesis. For smaller wave numbers, dimensional bounds on the structure functions emerge from the Navier-Stokes equation itself, confirming that velocity does indeed scale.

Self-Similarity

Most turbulent flows are not strictly self-similar. For example, turbulent bounded wall loaded flows (such as turbulent boundary layers, channel flows and pipe flows) exhibit several characteristic, large, flow-length scales. When the scales are presented in inner variables, one identifies the viscous length scale and momentum-scaleflow. In outer variables, the boundary-layer thickness, the pipe radius and the distance of the position where the velocity is evaluated from the wall are selected. Early studies of self-similarity in turbulent wall bounded flows were stimulated by the fact that the only scale at the wall is the viscous length scale and the momentum scale. In the early literature it has been postulated that the flow is self-similar if it can be described by only these scales. This early attempt is not quite satisfactory due to the fact that the effect of viscosity on the momentum-transfer is not confined to an infinitesimally thin slab at the wall. The momentum transfer in the center of the pipe is of course viscous, resulting in a mean velocity gradient. This does exclude the requirement of self-similarity. It follows that self-similar flows can be observed only in the very vicinity of the wall (ru+<30). Via a dimensional analysis by Barenblatt, it was shown that self-similarity of the flow in this region requires the introduction of a Wall variable W="u+/"ln ru+.

Real-Life Applications of Turbulence

Turbulence is a fundamental concept in the natural world whose influence and relevance extend across a vast range of spatial scales from biophysics, to atmospheric dynamics, to astrophysics. Realistic systems often strongly deviate from the paradigmatic case of simple passive fluids that are well modelled by the incompressible Navier-Stokes equations, and frequently involve additional internal drive and dissipation that take place on multiple spatial scales. A paradigmatic example are dense bacterial suspensions which around volume fractions of about 40% self-sustain a state of low-Reynolds-number turbulence. A minimal continuum model that incorporates essential additional features such as the resulting colloidal interactions, can be described by the incompressible Navier-Stokes equations supplemented by a generic Swift-Hohenberg term (Bratanov, 2015). The complex coupling to density variations that arises from bacterial motility and proliferation can be taken into account with an extended Toner-Tu theory that includes a weak symmetry-breaking field modelling polar alignment and captures quantitatively the behaviour of dilute suspensions. These concepts can also be combined for dense systems that feature both bacterial-motility induced density patterns and low-Reynolds-number turbulent dynamics. Combined analytical and numerical studies showed that the isotropic 2D power-law energy spectrum observed at small wave numbers with an exponent close to "8/3 can be obtained from the Swift-

Hohenberg framework and turns out to feature a pronounced non-universality. The slope depends on both the magnitude of nonlinear advection and the system-size, but converges for large domains when all relevant scales of the bacterial-velocity field are well separated. In addition, classical turbulence theories still successfully provide key information on the spatial structure of turbulent velocity fields and the corresponding form of the turbulent energy spectrum.

Engineering Applications

The design and maintenance of ships, aircraft, and various turbomachines depend upon the efficient and reliable operation of fluid systems. The features of these systems include complex geometry, unsteady flow, compressibility, rotation, shocks, boundary layers, and turbulence. Turbulent flow generally occurs at high Reynolds number, so turbulence typically exists throughout the entire flow field (even in the wake), thus greatly affecting overall system performance. Consequently, economizing on energy consumption, reducing environmental impact, and improving system structure all strongly depend on the effective and accurate prediction of turbulent flow.

The development of turbulence theories and models has been based on the initial pioneering contributions of Prandtl, Taylor, von Kármán, and Kolmogorov. Engineers typically use these theories to develop practical methods with which to deal with turbulent flows. Reynolds Averaged Navier–Stokes (RANS) computations and the Reynolds stress method can both be considered final versions of turbulent flow prediction models for practical application. However, the implementation of complex Reynolds stress models for turbulent flows that are neither steady nor homogeneous is still unavailable due to the huge difficulty of closures, resulting in the common practice of preferring simpler and more efficient methods that can process industrial problems reasonably well. In elaborating RANS procedures where all the complex mechanisms of turbulence are treated as a whole, the Navier–Stokes equations are rigorously averaged, with the stress tensor split into laminar and turbulent components. The averaged solution of turbulent flows in complex engineering problems involving combustion, fluid–structure interaction, heat transfer, and free surface flows is still a challenging task (A. Khossousi, 1987).

Astrophysical Implications

Astrophysical measurements commonly reveal inverse-cascade energy spectra with a k^{-2} scaling rather than the anticipated Kolmogorov/Kraichnan k^{-5/3} spectra. While an earlier study ascribed this anomaly to large-scale friction, the present analysis reproduces the k^{-2} scaling without friction, indicating the need to reconsider the conventional explanation (Ryan Westernacher-Schneider et al., 2015). Relativistic scaling relations have also been derived for two-dimensional conformal fluids in the weakly compressible regime, with numerical simulations confirming the validity of the scaling exponents predicted by the relevant correlation functions.

Meteorological Significance

Spectral slope differences and related exponents are linked to scaling properties, which are key for characterizing multifractals. The original hypothesis on turbulence, relating to a constant spectral slope, presupposed specific scaling relations among turbulent quantities, as evidenced in the law of energy distribution. A direct connection is also discernible for the so-called structure function in turbulence theory.

A different interpretation of the dimensional reasoning can be unified by considering the scalable nature of the cascade, which follows directly from the dimensional relations associated with the inertial range concept. The self-similarity characteristics of turbulence indeed rely on the existence of an inertial range in the energy cascade. Although other scaling relations appear plausible within the self-similarity framework, the energy distribution and inertial range scaling uniquely enable the calculation of the turbulence spectral slope. Further, the constancy of the inertial range supports the self-similarity concept when applied to relations involving other physical variables of the eddies.

Comparative Analysis of Turbulence Models

A model in which the N-dimensional space is decomposed into a Whitney sum of complexity-reducing fractal subsets, each of codimension not necessarily integer and each characterised by a suitable behaviour of the correlation functions, is analysed. The consideration of a spectrum of singularities revisits the well-known multifractal formalism. The multifractal model describes scaling properties of some fractal subset of the N-dimensional space. Instead, the present approach aims at describing scaling properties of all the points of the N-dimensional space, through the consideration of a multifractal distribution of scaling exponents. A justification for the new model stems from an analogy with dynamical systems. Multifractional functions are generated by noises with a wide spectra of singularities; in particular, such a case arises when the associated function is expressed as the sum of uncorrelated functions with different exponents.

An application of the idea of many-fractal scaling to turbulent flows is discussed. Homogeneous and isotropic turbulence is assumed. A one-power-law behaviour is posited for the scaling of longitudinal velocity increments on each subset C(h) of the N-dimensional space. The traditional scaling of the longitudinal velocity increments on the fractal set C(h) is, on the contrary, assumed with an r-dependent prefactor scaling as (r/L)N3-D(h). The behaviour of the longitudinal structure functions is consequently reproduced by the Ansatz . As the dimensionality approaches the space dimension=N, D(h) becomes the dimension of the fractal set C(h). Hence, the multifractal model is contained in the present approach as a special case. This many-fractal model appears able, in principle, to explain some puzzling results relative to the experimental determination of the intermittency correction to the Kolmogorov"41 power spectrum from passive scalars in turbulent shear flows and, more in general, to predict, for the flatness of the velocity field, a Reynolds-number scaling different from the usual two-thirds power of that for the energy dissipation field.

Direct Numerical Simulation (DNS)

The flow at high Reynolds numbers can be simulated with a direct numerical simulation (DNS) code that integrates the Navier-Stokes equations in their elliptic form without modeling assumptions. The full convective term is accounted for in such an approach, allowing the code—which is second order accurate in space and fourth order accurate in time—to capture significant scales of motion. Complementing the DNS experiments, a subgrid modeling strategy based on the enhanced subgrid-scale Green's tensor approach derives a vorticity–velocity formulation at moderate Reynolds numbers for two-dimensional unsteady incompressible flows. Large-eddy simulations of turbulent flows based on these models, which yield effective viscosities dependent on the filtered field, have been successfully integrated into such codes (Verstappen et al., 1994).

Large Eddy Simulation (LES)

Large eddy simulation (LES) represents a middle ground between Reynolds-averaged Navier–Stokes turbulence models and direct numerical simulation; it is a technique in computational aid to understanding turbulent flow. By calculating the time evolution in an unsteady flow field, LES explicitly computes large-scale motions, allowing smaller-scale motions to be modelled, usually with an eddy viscosity. Large eddy simulation methods find a compromise between high accuracy and great computational time of direct numerical simulation and the Reynolds-averaged Navier–Stokes equations (RANS) model, as they resolve only the large-scale turbulence. Recent applications of LES include studies of turbulent boundary-layer flow, and complex behavior in the flow over aircraft wings.

Reynolds-Averaged Navier-Stokes (RANS)

The RANS equations express a balance over a given spatial domain. Velocity and pressure are Reynolds decomposed, with the time averages denoted by overbars. The RANS equations are then obtained by substituting these relations into the incompressible NS equations, time averaging them, and assuming that temporal and spatial derivatives commute. The fluctuating velocity products make the RANS equations unclosed, which means additional physical assumptions (i.e., a turbulence closure model) are required to relate the Reynolds stress to the mean velocity. One such approach uses the Boussinesq hypothesis, which resembles the molecular viscous stress.

The Poisson equation for the fluctuating-pressure field is found by taking the divergence of the NS equation. The fluctuating-pressure term on the right-hand side is the source, physically representing the redistribution of turbulent energy among all turbulent-covariance components. The equation shows that fluctuating-pressure fluctuations arise to maintain a nearly divergence-free fluctuating velocity field.

Challenges in Turbulence Research

Common randomness or contagion. A related issue in the context of copulas and contagious jump processes is that no limitations on the dependence between jumps are imposed by the Lévy copula on the tail behaviour. For example, no restrictions on the dependence between the size of jumps in a two-dimensional. An important set of Brownian increments are the successive increments on the trajectory of a single classical Brownian motion. Such brownian increments or brownian shaping filters form the basis for a much larger class of processes, the increment processes.

Future Directions in Turbulence Studies

The phenomenology and forcing strategies discussed for active turbulence focus on the statistical properties of the steady-state spectral energy flux, which relates to the forcing by means of the energy-transfer-rate function. Generally, this relates to the statistics of the linear operator that is responsible for forcing. The linear instability mechanism transfers energy up the scale of the motion through the linear forcing spectrum. In contrast, stationary statistical representations of hydrodynamic turbulence typically consider only the stability properties of the steady-state energy distributions. Exploring the stability properties of the Gaussian spectral energy distributions of active turbulence provides intriguing insights into the mechanism that induces a reversal of the cascade direction, an observation noted in quasistationary forced states of two-dimensional hydrodynamic turbulence. Given that the Politis–Chlond scaling law for two-dimensional hydrodynamic turbulence has been rigorously established, it should be possible to perform a similar analysis for the Politis–Chlond active turbulence scaling relations derived in this study.

Conclusion

In summary, a turbulence model for active fluids with a fourth-order Landau-type velocity potential and an incompressible flow has been proposed. The resulting stress tensor generalizes the classical Navier-Stokes equation and reveals connections with turbulence models from a large-scale perspective.

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