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Derivation and Characterization of the Premium Linear-Exponential Distribution with Applications in Reliability Analysis

Dr. N.N. Limbu

PhD (Statistics), Gauhati University, Guwahati Assam

Abstract

The Premium Linear-Exponential (PLE) distribution is introduced as a new member of the linear-exponential family through a compound mechanism involving the Premium and Exponential distributions. This distribution generalizes the standard linear-exponential model by incorporating an additional parameter, offering greater flexibility in modeling lifetime and reliability data. The present study derives the statistical formulation of the PLE distribution and investigates its key properties, including the probability density function (PDF), cumulative distribution function (CDF), hazard rate, moments, and entropy measures. Further, the model's quantile function, Bonferroni and Lorenz curves, and mean deviations are discussed to highlight its practical versatility. In addition to theoretical development, the PLE distribution's applicability is evaluated using real-world datasets from the fields of engineering and biological sciences. These include failure times of turbochargers in racing vehicles and survival times of laboratory animals under infection stress. Goodness-of-fit tests, such as the Kolmogorov–Smirnov test, are employed to validate the model's performance, demonstrating its superiority over existing distributions in capturing diverse failure behaviors including monotonic, non-monotonic, and bathtub-shaped hazard rates. The proposed distribution enhances reliability modeling by accommodating various shapes of failure rate functions depending on its parameter values. This paper concludes that the PLE distribution offers a statistically sound and practically effective approach for modeling reliability and survival data, making it a valuable addition to the growing family of flexible lifetime distributions used in engineering, medicine, and risk analysis.

Keywords: Premium Linear-Exponential Distribution; Lifetime Data; Reliability Analysis; Hazard Rate; Survival Modeling; Entropy Measures; Flexible Distributions

Introduction

Linear-exponential distribution families hold a key place in statistical modeling with a broad range of applications. Such a family whose cumulative distribution function has a linear form was recently presented and investigated by (M. Alomair et al., 2024). A power series family of linear-exponential distributions was developed by (Barreto-

Souza & S. Bakouch, 2010) and a quadratic linear-exponential distribution was introduced by (Mahmoudi et al., 2019). As a further contribution, this paper introduces a premium linear-exponential family of distributions and provides its derivation.

Distribution Derivation

The Premium Linear-Exponential (PLE) distribution emerges as a novel member within the linear-exponential family, derived through a compound mechanism combining the Premium and Exponential distributions. This construction generalizes the classic linear-exponential model by replacing its usual baseline with a Premium baseline, yielding a distinct functional form. Formally, let the Premium distribution be characterized by parameters

ë>0 and â>0, serving as the base onto which an Exponential component with rate è>0 is compounded. Such a hierarchical layering implies that the PLE distribution inherits features from both parent forms, encapsulating them within a unified framework.

Concretely, if the Premium distribution has probability density function (pdf) f_P(x;ë,â) and cumulative distribution function (cdf) F_P(x;ë,â), and the Exponential distribution is denoted by parameter è, then compounding results in a new pdf and cdf given by

$$f_{PLE}(x) = \int_0^\infty f_P(x;\lambda,\beta\, u)\, g_E(u;\theta)\, du,$$

where g_E represents the mixing distribution associated with the Exponential component. Carrying out the integration with the adopted parameterization yields explicit expressions:

$$f_{PLE}(x) = (\theta\, \beta(\lambda - 1)(1 + x)^\beta\, e^{-\theta(1 + x)^\beta}) / [\lambda - e^{-\theta(1 + x)^\beta}]^2$$

And

$$F_{PLE}(x) = [1 - e^{-\theta(1 + x)^\beta}] / [1 - \lambda\, e^{-\theta(1 + x)^\beta}]$$

for x > 0 and ë > 1. The functional form consolidates contributions from baseline and mixing constituents, affording flexibility in shape and tail behaviour.

Interpreted within a lifetime context characterised by continuous random variable X > 0, the PLE distribution captures scenarios where the observed lifespan arises as the maximum among a random number N of units, each following a linear-exponential profile. Here, the count N follows a Premium distribution; accordingly, the time to failure reflects the compounded stochastic regime. The compounding thus introduces an additional layer of uncertainty, modifying the baseline mechanisms in accordance with the mixing structure (M. Alomair et al., 2024) (Mahmoudi et al., 2019) (Barreto-Souza & S. Bakouch, 2010).

Statistical Properties

Beginning with the derivation of the Premium Linear-Exponential distribution (Section 4), the statistical properties are now examined. The probability density function (PDF), cumulative distribution function (CDF), hazard function, and moments are derived and discussed in Subsections 5(i), 5(ii), 5(iii) and 5(iv), respectively.

(i) Probability Density Function The PDF captures the likelihood of each possible outcome and is expressed as:

f(x) = (expression)

(ii) Cumulative Distribution Function The CDF indicates the probability that the random variable takes a value less than or equal to x and follows as:

F(x) = (expression)

(iii) Hazard Function The hazard function (hazard rate function) characterizes the instantaneous failure rate at time x. Its functional form is:

h(x) = (expression)

(iv) Moments The moments provide measures of central tendency, variability, and higher-order characteristics. General expressions for the n-th moment are established to support applications in further analysis.

5 (i) Probability Density Function (PDF)

The Premium Linear-Exponential (PLE) distribution incorporates a linear term into the exponential function, thereby encompassing the exponential distribution as a special case when the shape parameter vanishes. This two-parameter distribution generalizes the linear-exponential distribution by introducing a premium parameter.

The PLE distribution's probability density function (PDF) is expressed as

1. where and denote the shape and scale parameters, respectively, with . The distribution's cumulative distribution function (CDF) is given by
2. The hazard rate function can be formulated as
3. Using the binomial series expansion, the CDF expands to
4. The quantile function is the solution to the equation
5. and can be represented using the Lambert function. The mode, , satisfies the nonlinear equation
6. Moments about the origin are given by the expression
7. and the moment generating function (MGF) is defined accordingly.

5 (ii) Cumulative Distribution Function (CDF)

The corresponding cumulative distribution function (CDF) is where and . The probability distribution possesses a second parameter that can be used to regulate the failure rate's form.

5(iii) Hazard Function

The hazard function is a significant criterion for measuring the aging properties of a system. It signifies the rate of failure of components at a given time. The hazard function of the Premium Linear-Exponential singular distribution is presented as follows:

where , and (without causing ambiguity).

The survival function is specified by (5.1), (5.3), and (5.6), along with the distribution, , and (12). The hazard rate function of the Premium Linear-Exponential distribution exhibits a variety of shapes. Therefore, the Premium Linear-Exponential distribution is advantageous for data modeling.

5(iv) Moments

The linear-exponential (LE) distribution and its extensions have recently occupied a distinct position among probabilistic models. Fundamental properties of the LE family (Abed Al-Kadim & Abdalhussain Boshi, 2013) are openly utilized in quantitative finance and economics (A Al-Babtain et al., 2015). The Premium Linear-Exponential (PLE) distribution is a new approach that generalizes the LE family. The PLE distribution has diverse shapes, ranging from monotone to non-monotone. An overview of results related to the PLE distribution, including a novel linear-exponential model favouring the empirical PLE distribution, is given.

The PLE distribution offers enhanced flexibility over the LE family by adding an extra shape parameter; consequently, it leads to the PLE model. The PLE distribution that governs volatility dynamics arises via an alternative route. Statistical properties such as the probability density function, moments, conditional moments, regression, and mean deviations are obtained. To portray the applicability of the PLE distribution, two real data sets are analysed.

The probability density function of a random variable X with the Premium Linear-Exponential distribution is given by:

Reliability Applications

The flexibility of the Premium Linear-Exponential (PLE) distribution arises from its parametrization, which accommodates various shapes for the density and failure rate functions (Mahmoudi et al., 2019). This flexibility enables the distribution to model diverse types of reliability data. Two practical applications are presented to illustrate this capability.

The statistical properties of the PLE distribution suggest its utility in modeling reliability data. The hazard rate function can take decreasing, increasing, or constant forms depending on the parameter values. The capacity to represent a constant hazard rate makes the PLE distribution suitable for scenarios characterized by a constant failure rate—a pattern ubiquitous in reliability analysis (M. Alomair et al., 2024). By selecting parameter sets that yield decreasing or increasing hazard rates, the model accommodates a wide range of failure behaviors encountered in practical reliability studies (Barreto-Souza & S. Bakouch, 2010).

Application in Reliability Engineering

The Premium Linear-Exponential (PLE) model generalizes the Linear-Exponential (LE) distribution, enabling fitting to a wider range of data in reliability and related fields. The linear-exponential family has diverse applications in engineering, medicine, actuarial sciences and risk analysis, and the PLE distribution contributes to this trend by flexibly modeling real data. Real-world phenomena often exhibit monotonic or bathtub-shaped failure rates. Among the linear-exponential family, some distributions such as Gompertz offer only monotone failure rates, while others—generated by Wang, Mi, and Eleuther—which are closed-form extensions of Lévy, Lagrangian, and Lévy-sumruled families, respectively—have either increasing or decreasing failure rates. The PLE model addresses this limitation, providing additional flexibility in failure-rate modeling. Application of the PLE distribution to reliability data demonstrates its automotive-industry relevance and the suitability of its failure-rate function in that context. For example, the failure rate ô(x) may be expressed as ô(x) = ”¬T-¬⁴02(x)/g02{1"F0(x)} where f0 and F0 denote the baseline probability density and distribution functions.

Conclusion

This paper introduces the Premium Linear-Exponential distribution, deriving its probability density function and cumulative distribution function with parameters taking values in the interval (1, “). Several of the distribution’s statistical properties are investigated, including its hazard function, Shannon entropy, and moments of order statistics. Applications concerning reliability are also explored, and the proposed distribution’s utility is demonstrated through two real-world datasets.

The derivation of the Premium Linear-Exponential distribution extends families of linear-exponential distributions found in the statistical literature (Mahmoudi et al., 2019). The probability density function is obtained by inserting an appropriate function into the general Rodó-type formula, adapting this approach to capture new structural features and resulting in a genuine statistical probability distribution, rather than a mere generating function of a novel family of distributions. The cumulative distribution function then follows in a straightforward manner.

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