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An Inventory Model with Partial Backlog for Declining Items with Varying Selling and Purchasing Prices

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Abstract

Selling price and purchase cost typically vary with economic conditions in today's competitive markets. For a business to be profitable, both the selling price and the purchasing cost are essential. As a result, I expand the inventory model Teng and Yang (2004) established in this study to incorporate the ability for both the selling price and the purchasing cost to fluctuate over the course of a defined time horizon between replenishment cycles. The goal is to determine the best pricing strategy and replenishment schedule in order to maximise profit. The existence, uniqueness, and global optimality of the suggested solution are guaranteed by the conditions that lead to a maximising solution. Some theoretical results and an effective approach for solving the problem are offered. Finally,numerical examples for illustration and sensitivity analysis for managerial decision making arealso performed.

Keywords- Partial backlog , inventory, deteriorating item, varying price

1. Introduction

In today's time-based competitive market, a product's unit selling price may rise significantly in response to an increase in demand, particularly for costly or popular commodities. Yet, because to things like competition, technological advancements, and other factors, the selling prices of goods can decrease significantly over the course of their life cycles. As a result, the selling price is not constant. On the other hand, certain products see a reduction in cost as demand rises. For example, the unit cost of a high-tech product experiences a large decline during the course of its brief product life cycle.

For example, the cost of a personal computer drops constantly as shown in Lee et al. 1.

Additionally, as Heizer and Render 2 have noted, the purchase cost as a percentage of sales is frequently significant. Thus, it is crucial to consider the fluctuating selling price and purchase cost from the standpoint of integrated logistics management.

Furthermore, in actuality, the backorder rate decreases with the duration of the waiting period for high-tech and trendy commodities with short life cycles. When a provider runs out of stock, customers are less inclined to repurchase from them and are more likely to shop elsewhere. A shift in customer tastes or the arrival of more competitive products could result in a fall in the product's sales. The backlogging rate decreases as waiting times increase. As a result, there is less profit and a higher percentage of lost transactions.

Therefore, it is essential to consider the partial backlog issue. Abad [3] suggested the best price and lot-sizing strategy for situations including partial backordering and perishability. The partial backlog inventory model with time-varying demand and purchasing cost was examined by Teng et al. [4]. After taking into account the inventory model with selling price and purchasing cost, Chang et al. [5] offered the best replenishment strategy for a retailer to reach maximum profit. Various models were created by numerous authors to investigate the impact of the elements (buying cost and/or selling price) on the relevant issue.For instance, under generalised holding costs, Teng and Yang [6] presented inventory lot-size models with time-varying demand and purchasing costs. Abad [7] took the pricing strategy into account and gave a retailer the best price and lot size when the selling price determined the demand. Sana [8] recently presented the best pricing strategy for an inventory model with partial backlogs and price-dependent demand. An ideal shipment plan for defective items in a stock-out scenario was put out by Das Roy et al. [9]. Once more, Das Roy et al. [10] offered an economic-order quantity model for incomplete backlog items of unsatisfactory quality.The major assumptions and objectiveused in the above research articles are summarized in Table 1.

Therefore, in contrast to the previously stated publications, the inventory model used here was created as proposed by Teng and Yang [11] to account for changing purchasing and selling costs that vary across a finite time horizon from cycle to cycle of replenishment. Finding the best pricing strategy and replenishment schedule is the goal, not minimising costs, but rather maximising profit.

Because the overall profit from the inventory system is a concave function of the number of replenishments, finding the ideal number of replenishments to reach a local maximum ismade easier. Additionally, a naturally derived approximation for determining the ideal refill quantity is offered. Sensitivity analysis for managerial decision making is carried out, along with some numerical examples for illustration. A summary and recommendations for additional research are given at the end.

2. Assumptions and Notation

The following presumptions form the foundation of the inventory replenishment problem's mathematical model:

1. Here, the inventory problem has a finite planning horizon, which is represented by H time units. During the time horizon H, the initial and final inventory

levels are both 0.

2. There is no lag time and quick replenishment.
3. In actuality, the object might deteriorate with time. For the sake of simplicity, we'll assume that there is no repair or replacement for the degraded objects, and that the rate of deterioration is constant.

Table 1: fundamental feature of inventory models on certain articles.

Author(s)(published year)	Demand rate	Deterioration rate	Allow for shortages	With partial backlogging	Purchasing cost	Selling price	Objective
Abad (1996)[3]	Price dependent	Time varying	Yes	Yes	Constant	Variable	Profit maximization
Teng et al. (2002)[4]	Time dependent (logconcave)	Constant	Yes	Yes	Constant	X	Cost minimization
Teng and Yang 2004[11]	Time dependent	Constant	Yes	Yes	Timevaryin g	X	Cost minimization
Chang et al. (2006)[5]	Time and price dependent	Constant	Yes	Yes	Constant	Variable	Profit maximization
Teng and Yang (2007) [6]	Time dependent	X	Yes	X	Timevaryin g	X	Costminimization
Abad (2008)[7]	Price dependent	Time varying	Yes	Yes	Constant	Variable	Profit maximization
Sana (2010)[8]	Price dependent	Time varying	Yes	Yes	Constant	Variable	Profit maximization
Roy et al. (2011a)[9]	Constant	UniformDistr ibution	Yes	Yes	Constant	Constant	Costminimization/Pr ofitmaximization
Roy et al. (2011b)[10]	Constant	UniformDistr ibution	Yes	Yes	Constant	Constant	Profit maximization
Present paper	Time dependent	Constant	Yes	Yes	Timevaryin g	Timevar ying	Profit maximization

4. Shortages are allowed. Unsatisfied demand is backlogged, and the fraction of shortagesbackordered is a decreasing function of time t, denoted by ât, where t is thewaiting time up to the next replenishment, and 0 d"R"XÆ"⁴ ât d"R"XÆ"⁴ 1 with â0 1. Note thatif ât 1 or 0 for all t, then shortages are completely backlogged or lost.
5. In the scenario of lost sales, the opportunity cost includes both the revenue loss and the cost of lost goodwill. Therefore, the opportunity cost in this situation is higher than the unit purchasing cost. For details, see Teng et al. [4].
6. In today's highly competitive global market, we assume that both the selling price and purchasing cost vary over time and change from one replenishment cycle to the next within a finite time frame.

For convenience, the following notation is used throughout this paper:

H: the time horizon under consideration,

$f(t)$: the demand rate at time t, without loss of generality, we here assume that $f(t)$is increase, positive, differentiable in [0,H]

$C_v(t)$: the purchasing cost per unit at time t, which is positive, differentiable in [0,H],

p(t) : the selling price per unit at time t, which is positive, differentiable in [0, H],

è: the deterioration rate,

c_f : the fixed ordering cost per order,

c_h: the inventory holding cost per unit per unit time,

c_b: the backlogging cost per unit per unit time, if the shortage is backlogged,

c_l: the unit opportunity cost of lost sales, if the shortage is lost. We assume without loss of generality that $c_l > c_v(t)$,

n: the number of replenishments over [0, H] a decision variable,

t_i: the ith replenishment time a decision variable, i 1, 2, . . . , n,

s_i: the time at which the inventory level reaches zero in the ith replenishment cycle a decision variable, i 1, 2, . . . , n.

4. Mathematical Model

For simplicity, we use the same inventory model as in Teng and Yang 11, which is shown inFigure 1.

As a result. we obtain the time-weighted inventory during the ith cycle as

$$I_i = \frac{1}{\theta} \int_{t_i}^{s_i} \left[e^{\theta(t-t_i)} - 1 \right] f(t)\, dt, \quad i \; 1, 2, \ldots, n. \qquad (3.1)$$

Similarlv. the time-weighted backorders due to shortages during the ith cycle is

$$B_i = \int_{s_{i-1}}^{t_i} (t_{i-1}) \hat{a} (t_{i-1}) f(t)\, dt, i = 1, 2, \ldots, n, \qquad (3.2)$$

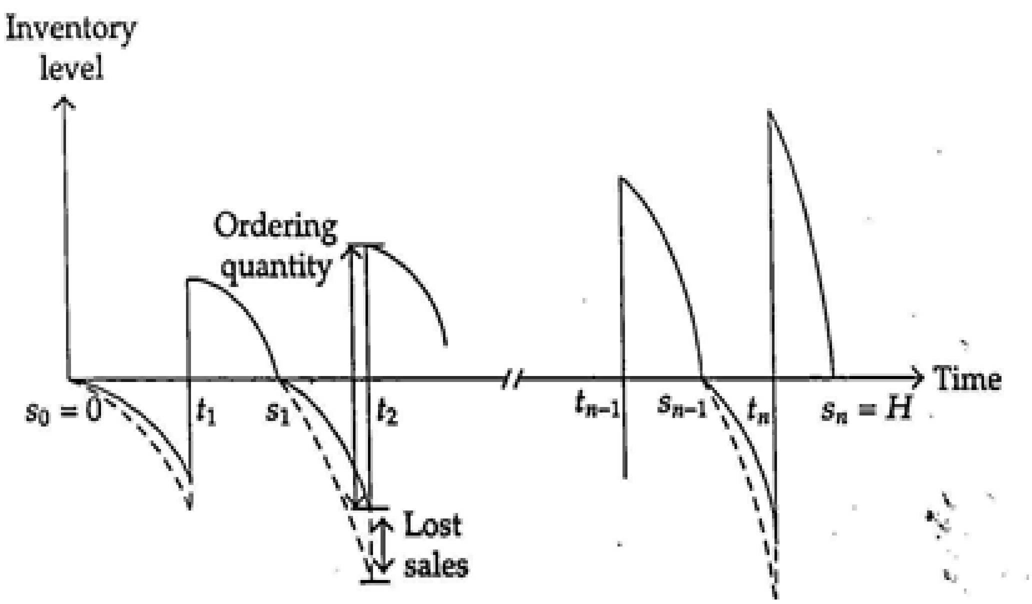

Figure 1: Graphical representation of inventory model.

and the total number of lost sales due to shortages during the ith cycle is

$$L_i = \int_{s_{i-1}}^{t_i} [1 - \beta(t_i - t)] f(t)\, dt,\ i = 1, 2, \ldots, n, \quad (3.3)$$

The order quantity and unit sold at t_i in the ith replenishment cycle is

$$Q_i = \int_{s_{i-1}}^{t_i} \beta(t_i - t) f(t)\, dt + \int_{t_i}^{s_i} e^{\theta(t-t_i)} f(t)\, dt, i = 1, 2, \ldots, n, \quad (3.4)$$

and unit sold at t_i in the ith replenishment cycle is

$$S_i = \int_{s_{i-1}}^{t_i} \beta(t_i - t) f(t)\, dt + \int_{t_i}^{s_i} f(t)\, dt, i = 1, 2, \ldots, n, \quad (3.5)$$

Therefore, the purchasing cost during the ith replenishment cycle is

$$P_i = c_f + c_v(t_i)\, Q_i$$

$$= c_f + c_v(t_i)\left[\int_{s_{i-1}}^{t_i} \beta(t_i - t) f(t)\, dt + \int_{t_i}^{s_i} e^{\theta(t-t_i)} f(t)\, dt\right] \quad (3.6)$$

Therefore, the purchasing cost during the ith replenishment cycle is

$$R_i = p(t_i)\left[\int_{s_{i-1}}^{t_i} \beta(t_i - t) f(t)\, dt + \int_{t_i}^{s_i} f(t)\, dt\right] i = 1, 2, \ldots, n, \quad (3.7)$$

Hence, if n replenishment orders are placed in [O,H], then the total realized profit of the inventory system during the planning horizon H is

$$TP\ (n, \{s_i\}, \{t_i\}) = \sum_{i=1}^{N} (R_i - P_i - C_h I_i - c_b B_i - c_t L_t)$$

$$= \sum_{i=1}^{N} \int_{s_{i-1}}^{t_i} \{[p(t_i) - c_V(t_i - t) + c_l]\ \beta(t_i - t) - c_l\} f(t)\, dt - nc_f\} \quad (3.8)$$

$$+ \sum_{i=1}^{N} \int_{t_i}^{s_i} \left[p(t_i) - c_V(t_i) - \left(\frac{c_h}{\theta} + c_V(t_i)(e^{\theta(t-t_i)} - 1)\right]f(t)\, dt$$

with 0 $s_0 < t_1$ and $s_n = H$. The problem is to determine n, $\{s_i\}$, and $\{t_i\}$ such that

TP(n, $\{s_i\}$, $\{t_i\}$ in (3.8) is maximized.

4. Theoretical Results and Solution

For a fixed value of n, the necessary conditions for TP(n,{si}, {ti} to be maximized are:∂TP(n, {si}, {ti}/∂ti= 0 for i 1, 2, . . . , n, and ∂TP(n, {si}, {ti}/∂si= 0, for. i =1, 2, . . . , n − 1.Consequently, we obtain

$$\int_{t_i}^{s_i} \{p^l(t_i) + [c_h + \theta\, c_V(t_i) - c_v^l(t_i)]e^{\theta(t-t_i)}\}f(t)\,dt$$

$$= \int_{t_i}^{s_i} \{[p^l(t_i) - c_v^l(ti) - c_b]\,\beta(t_i - t) + [p^l(t_i) + c_l - c_V(t_i - t)\beta(t_i - t)\}f(t)\,dt \quad (4.1)$$

$$[p(t_{i+1}) - c_V(t_{i+1}) - c_b(t_{i+1} - s_i)]\,\beta(t_{i+1} - s_i) - c_i[1 - \beta(t_{i+1} - s_i)$$

$$= p(t_i) - c_V(t_i) - \left[\,(\tfrac{c_h}{\theta} + c_V(t_i)\right](e^{\theta(t-t_i)} - 1) \quad (4.2)$$

Respectively. Note that (4.1) and (4.2) are coincident with the following articles'

(1) Equations (12) and (11) in Teng and Yang [11], if p(t) = 0,

(2) Equations (15) and (14) in Teng et al. [4], if p(t)= 0 and $c_v(t)= c_v$,

(3) Equations (11) and (10) in Chang et al. [5], if p(t) = p and $c_v(t) = c_v$.

Thus, the model here proposed is a generalization of the above three mentioned models. For simplicity, from (4.2), let the marginal resultant profit per unit during no shortage and shortage period be

$$R\,(t, u) = p(t) - c_v(t) - [\tfrac{c_h}{\theta} + c_v(t)]\,(e^{\theta(t-t_i)} - 1) \quad (4.3)$$

with $t \leq u$, and

$$P(s, t) = [p(t) - c_v(t) - c_b(t- s)]\beta\,(t- s) - c_i\,[1- \beta(t - s)] \quad (4.4)$$

With $s \le t$, respectively. Taking the partial derivative of R(t, u) and P(s,t) with respect to t respectively, we obtained the following results:

$$R_t(t,u) = p(t)+ [ch+\theta c_v(t) - c_v(t)\ e^{\theta(u-t)}, \quad (4.5)$$

$$P_t(s,t)= [p(t) - c_v(t) - c_b]\beta (t - s) + [p(t) + c_l - c_v (t) - c_b(t - s)\ \beta (t - s). \quad (4.6)$$

Note that the longer the waiting time, the lower the marginal resultant profit. Consequently, P(s,t) is a decreasing function of t. Thus, we may assume without loss of generality that $P_t (s,t) < 0$, for all $t > s$. Then, we obtain the following result.

Lemma 4.1. For any given n, if $R_{ti} (t_i,t) \le 0$, with $t \ge t_i$, and $P_t(i,t)$, $ti < 0$, with $t \le ti$, i =1, 2, . . . , n, then the optimal solution is $n* = 1$ and $t{*}t_1^* = 0$ (i.e., purchase at the beginning).

Proof See Appendix A.

The results in Lemma 4.1 can be interpreted as follows. The condition $R_{ti} (t_i, t) \le 0$implies that $p (t)+ [c_h + \theta c_v(t)]e^{\theta u-t} \le c_v(t)\ e^{\theta u-t}$. This means that the increasing rate of the unit purchasing cost is higher than or equal to the sum of the marginal selling price and marginal inventory carrying cost per unit which includes inventory and deterioration costs.

Therefore, buying and storing a unit and then selling now are more profitable than buying and selling it later.

Theorem 4.2. For any given n, if $R_{ti}(t_i, t) > 0$, with $t \ge t_i$, and $Pti (t, t_i) < 0$, with $t \le t_i$, i =1, 2, . . . , n, then the solution that satisfies the system of (4.1) and (4.2) exists uniquely and $0 \le s_{i-1} < t_i < s_i$, for i =1, 2, . . . , n.

Proof. See Appendix B.

The result in Theorem 4.2 reduces the 2n-dimensional problem of finding $\{s_1^*\}$ and $\{t_i^*\}$to a one-dimensional problem. Since s_0= 0, we only need to find $\{t_1^*\}$ to generate s_1^* by (4.1), $t_2^*$by (4.2), and then the rest of $\{s_1^*\}$ and $\{t_i^*\}$ uniquely by repeatedly using (4.1) and (4.2). For any chosen t_i^*, if s_n^*= H, then t_1^* is chosen correctly. Otherwise, we can easily find the optimal$t_1^*$by standard search techniques.

Having calculated the second partial derivatives of the function TP (n, $\{s_i\}$, $\{t_i\}$ shows that the Hessian matrix is negative definite if

$$\frac{\partial^2 TP}{\partial t_i^2} \leq -\left[\frac{\partial^2 TP}{\partial s_i \partial t_i} + \frac{\partial^2 TP}{\partial t_i \partial s_{i-1}}\right] < 0, \text{ for i } 1, 2, \ldots, n, \qquad (4.7)$$

$$\frac{\partial^2 TP}{\partial s_i^2} \leq -\left[\frac{\partial^2 TP}{\partial s_i \partial t_i} + \frac{\partial^2 TP}{\partial t_i \partial s_{i+1}}\right] < 0, \text{ for i } 1, 2, \ldots, n, \qquad (4.8)$$

Theorem 4.3. For any given n, if $R_{ti}(t_i, t) > 0$, with $t \geq t_i$, and $P_{ti}(t, t_i) < 0$, with $t \leq t_i$, i = 1, 2, . . . , n, under conditions (4.7) – (4.8), then the solution that satisfies the system of (4.1) and (4.2) is a global maximum solution.

Proof. See Appendix C.

Next, we show that the total profit TP (n,$\{s_i^*\}$, $\{t_i^*\}$ is a concave function of the numberof replenishments. As a result, the search for the optimal replenishment number, n^*, is reduced to find a local maximum. For simplicity, let

$$TP(n) = P(n, \{s_i^*\}, \{t_i^*\}) \qquad (4.9)$$

By applying Bellman's principle of optimality [12], we have the following theorem:

Theorem 4.4.TP (n) is concave in n.

The result in Theorem 4.2 reduces the 2n-dimensional problem of finding $\{s_1^*\}$ and $\{t_i^*\}$ to a one-dimensional problem. Since $s_0 = 0$, we only need to find $\{t_1^*\}$ to generate s_1^* by (4.1), t_2^* by (4.2), and then the rest of $\{s_1^*\}$ and $\{t_i^*\}$ uniquely by repeatedly using (4.1) and (4.2). For any chosen t_i^*, if $s_n^* = H$, then t_1^* is chosen correctly. Otherwise, we can easily find the optimal t_1^* by standard search techniques.

Having calculated the second partial derivatives of the function TP (n, $\{s_i\}$, $\{t_i\}$ shows that the Hessian matrix is negative definite if

$$\frac{\partial^2 TP}{\partial t_i^2} \leq -\left[\frac{\partial^2 TP}{\partial s_i\, \partial t_i} + \frac{\partial^2 TP}{\partial t_i\, \partial s_{i-1}}\right] < 0, \text{ for i } 1, 2, \ldots, n, \qquad (4.7)$$

$$\frac{\partial^2 TP}{\partial s_i^2} \leq -\left[\frac{\partial^2 TP}{\partial s_i\, \partial t_i} + \frac{\partial^2 TP}{\partial t_i\, \partial s_{i+1}}\right] < 0, \text{ for i } 1, 2, \ldots, n, \qquad (4.8)$$

Theorem 4.3. For any given n, if $R_{ti}(t_i, t) > 0$, with $t \geq t_i$, and $P_{ti}(t, t_i) < 0$, with $t \leq t_i$, $i = 1, 2, \ldots, n$, under conditions (4.7) – (4.8), then the solution that satisfies the system of (4.1) and (4.2) is a global maximum solution.

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Proof. The proof is similar to that of Teng and Yang [11], the reader can easily prove it.

By a similar discussion as in Teng and Yang [11], I here use the average backlogging rate β, unit purchasing cost c_v and average unit selling price p to replace β (t_{i+1} −s_i), c_v (t_i) and p (t_i), respectively. The estimate of the number of replenishments is obtained as

$$n_1 = \text{rounded integer of}\left[\frac{(c_h+\theta c_v)[\,c_b\beta+(c_i-c_v+p)(1-\beta)HQ(H)]}{2c_f[c_h+\theta c_v + c_b\beta+(c_i-c_v+p)(1-\beta)]}\right]^{1/2}, \quad (4.10)$$

Where $Q(H) = \int_0^H f(t)\,dt$. It is obvious that searching for n^* by starting with n in (4.10) will speed the computational efficiency significantly, comparing to starting with n = 1.The algorithm for determining the optimal number of replenishments n^* and schedule I summarized as follows.

Algorithm for Finding Optimal Number and Schedule

Step 1. Choose two initial trial values of n^*, say n as in (4.10) and n − 1. Use a standard search method to obtain $\{t_i^*\}$ and $\{s_i^*\}$, and compute the corresponding TP(n) and TP(n − 1), respectively.

Step 2. If TP(n) ≥ TP(n − 1), then compute TP(n+1), TP(n+ 2), . . ., until we find TP(k)>TP(k+1). Set n^*= k and stop.

Step 3. If TP(n)< TP(n − 1), then compute TP(n − 2), TP(n − 3), . . ., until we find TP(k)>TP(k− 1. Set n^*= k and stop.

5. Numerical Examples

Example 5.1. Let f (t)= 200+ 20t, H = 3, p(t) = 200 + 30t, $c_v(t)$=150+10t, c_f= 250, c_h= 40,c_b= 50, c_l= 200, θ=0.08, β(t)=$e^{-0.4t}$ in appropriate units. After calculation, we have p = 245, c_v= 165, and â= 0.582. By (4.10), we obtain the estimate number of replenishmentsn_1 = 12. From computational results, we have TP(13) 49021.79, TP(14)= 49044.31, and TP(15)= 49030.61. Therefore, the optimal number of replenishments is 14, and the optimal profit is 49044.31. The optimal replenishment schedule is shown in Table 2.

Table 2: The optimal replenishment schedule for Example 5.1.

i	1	2	3	4	5	6	7
t_i	0.0849	0.3160	0.5437	0.7683	0.9899	1.2086	1.4245
s_i	0.2330	0.4626	0.6889	0.9121	1.1323	1.3497	1.5644
$p(t_i)$	202.55	209.48	216.31	223.05	229.70	236.26	242.73
$c_v(t_i)$	150.85	153.16	155.44	157.68	159.90	162.09	164.24
i	8	9	10	11	12	13	14
t_i	1.6378	1.8485	2.0569	2.2629	2.4666	2.6682	2.8676
s_i	1.7765	1.9861	2.1933	2.3982	2.6009	2.8015	3.0000
$p(t_i)$	249.13	255.46	261.71	267.89	274.00	280.05	286.03
$c_v(t_i)$	166.38	168.49	170.57	172.63	174.67	176.68	178.68

Table 3: Sensitivity analysis on parameters changed for Example 5.2

Parameter	Parameter value	% change in parameter	Estimated n_1	Optimal n^*	$TP^*(n^*)$	%change in$TP^*(n^*)$
c_f	200	−20	14	16	49787.47	1.52
	300	+20	11	13		−1.37

c_h	30 50	−25 +25	12 13	14 15	49379.80 48748.20	0.68 −0.60
c_b	40 60	−20 +20	12 13	14 14	49135.56 48961.05	0.19 −0.17
cl	150 250	−25 +25	12 13	14 14	49238.33 48883.42	0.40 −0.33
θ	0.06 0.1	−25 +25	12 13	14 14	49152.16	0.22 −0.21

Example 5.2. To understand the effect of changes in parameters c_f, c_h, c_b, c_l, è, â(t) on the optimal solution, the sensitivity analysis is performed by changing one parameter at a time and keeps the others unchanged. The parameter values are the same as in Example 5.1. The results obtained are shown as in Table 3.

From Table 3, the following phenomena can be obtained.

1. The optimal maximum profit decreases as c_f ,c_h, c_b, c_l, è increases, however, it increases as the backlogging rate â (t) increases.
2. The optimal maximum profit is more sensitive on parameters c_f than others.
3. The optimal replenishment number is very slightly sensitive to the change of the separameters except c_f and â(t).
4. The estimated number n1 is very close to the optimal replenishment number n", no matter what magnitude of the parameters changed.

Example 5.3. Using the same numerical values as in Example 5.1, we consider the influence of changes of the rate of change of selling price p(t) and purchasing cost c_v(t) on the total profit. The results are obtained as shown in Table 4.

Table 4: Sensitivity analysis on rate of change for Example 5.3.

p(t), c_v(t)	% change in rate of change	Estimated n_1	Optimal n^*	$TP^*(n^*)$	% change in $TP^*(n^*)$
p(t) 200 + 30t c_v(t)150+10t	— —	12	14	49044.31	0.00
p(t) 200 + 45t c_v(t) 150+10t	+50 —	13	15	64887.92	32.30
p(t) 200+15t c_v(t)	−50 —	12	14	33314.34	32.07

$c_v(t)$ 150+10t					
p(t) 200 + 30t cv(t) 150+15t	— —50	12	14	43768.8	−10.76
p(t) 200+ 30t cv(t) 150 +5t	— −50	12	14	54329.55	10.78

From Table 4, it is obviously that the phenomena are obtained.

1. The percentage change in total maximum profit is significantly sensitive on the variation of rate of change.
2. The total profit increases as the rate of change of selling price increases, while decreases as the rate of change of purchasing cost increases.
3. The estimated number n_1 is also close to the optimal replenishment number n''.
4. The optimal replenishment number is slightly sensitive to the change of rate of change.

6. Conclusions

This study examines a partial-backlogging inventory lot-size model for degrading goods with variable purchase costs, time-dependent demand, and selling prices. We demonstrate that there is only one ideal replenishment schedule and that the inventory system's overall profit is a concave function of the number of replenishments. A heuristic approximation for determining the ideal replacement quantity is offered. According to the results of the sensitivity analysis, there is a considerable impact on the behaviour of the system from variations in the rate of change of the purchasing cost and selling price.

The selling price and the purchase cost must therefore be included in the inventory model; this is especially important given the volatile nature of the current market. By adding more useful features, the model developed here can be expanded even further by adding new functions or parameters, like taking into account demand as a function of selling price or stock dependent, or time-varying deterioration rate. It can also be developed by adding additional factors, like inflation and price discount.

A. Proof of Lemma 4.1

Let

$$TPi(s_{i-1}, t_i, s_i t_i s_i) = \int_{s_{i-1}}^{t_i} \left\{ p(t_i) - c_v(ti) - c_i] \beta(t_i - t) - c_i \right\} f(t)\,dt$$

$$+ \int_{t_i}^{s_i} \left\{ p(t_i) - c_v(ti) - [\frac{c_h}{\theta} + cv(t_i)] (e^{\theta(t-t_i)} - 1) \right\} f(t)\,dt \quad (A.1)$$

$$= \int_{s_{i-1}}^{t_i} p(t, t_i) \int_{t_i}^{s_i} R(t_i, t)\, f(t)\,dt$$

We then have

$$\frac{\partial TP_i}{\partial t_i} = \int_{s_{i-1}}^{t_i} p_{ti}(t, t_i) f(t)\,dt + \int_{t_i}^{s_i} R_{ti}(t_i, t)\, f(t)\,dt. \quad (A.2)$$

If $R_{ti}(ti, t) \leq 0$, then we know from (A.2) that $\partial TP_i/\partial t_i \leq 0$. Therefore, for any given i, TP_i is decreasing with t_i. This implies that $TP_i(s_{i-1}, s_{i-1}, s_i) \geq TP_i(s_{i-1}, t_i, s_i)$ for any fixed i. Consequently, we obtain

$$TP\ (n, \{s_i\}, \{t_i\} = \sum_{i=1}^{N} TPi(s_{i-1}, t_i, s_i) - nc_f$$

$$= \sum_{i=1}^{N} TPi(s_{i-1}, t_i, s_i) - nc_f \quad == \sum_{i=1}^{N} \int_{s_{i-1}}^{s_i} R(s_{i-1}, t)\, f(t)\,dt - ncf (A.3)$$

$$\leq \sum_{i=1}^{N} \int_{s_{i-1}}^{s_i} R(O, t)\, f(t)\,dt - ncf\ (\text{since} R_{ti}(t_i, t) \leq 0\ )$$

$$\leq \int_{s_{i-1}}^{s_i} R(O, t)\, f(t) dt - cf$$

This completes the proof.

B. Proof of Theorem 4.2

For any given si−1 and ti, from (4.1), we set

$$F(x)=\int_{t_i}^{x} \{p^l(t_i) + [c_h + \theta c_v(ti) - c_v^l(t_i)]e^{\theta(t-t_i)}\}f(t)\,dt$$

$$+\int_{s_{i-1}}^{t_i} \{[p^l(t_i) - c_v^l(t_i) - c_h]\beta(t_i - t) + [p(t_i) + c_l - c_v(t_i) -$$

$$c_h(t_i - t)\beta(t_i - t)\}f(t)\,dt$$

$=\int_{t_i}^{x} Rt_i(t_i, t)f(t)dt + \int_{s_{i-1}}^{t_i} P_{ti}(t, t_i)\, f(t)\, dt$ *with* $x \geq t_i \geq s_{i-1}$ (B.1) we then have

$$F(t_i)=\int_{s_{i-1}}^{t_i} P_{ti}(t, t_i)\, f(t)\, dt < 0, \quad (B.2)$$

And $\lim_{x\to\infty} F(x) > 0$. Taking the first derivatives of F (x) with respect to x, we obtain

$$F^1(x)R_{ti}(t_i, x)\, f(x) > 0. \quad (B.3)$$

As a result, we know that there exists a unique si($>t_i$) such that $F(s_i)$ 0. Thus, the solution to (4.1) uniquely exists. Similarly, from (4.2), we set

$$G(x)\ [p(x) - c_v(x) - c_h(x - s_i) + c_l]\beta\ (x - si) - [p(t_i) - c_v(t_i) + c_l]$$

$+\left[\frac{c_h}{\theta} + c_v(t_i)\right](e^{\theta(t-t_i)} - 1) = p(s_i, x) - R(s_i, s_i)$, with $x \geq s_i \geq t_i$ (B.4)

We then have

$$G(s_i) = p(s_i) - c_v(s_i) - R(t_i, s_i) - R(s_i, s_i) - R(t_i, s_i) > 0, \quad (B.5)$$

Since $R_{ti}(t_i, t) > 0$, and $\lim_{x\to\infty} G(x) = -c_l < 0$. By taking the first derivatives of G(x) with respect to x, we obtain

$$G(x) = P_x(s_i, x) < 0. \quad (B.6)$$

Consequently, there exists a unique $t_{i1}(>s_i)$ such that G $(t_{i+1})=0$, which implies that solution to (4.2) uniquely exists. Therefore, we complete the proof.

C. Proof of Theorem 4.3

Taking the second derivatives with respect to t_i and s_i on TP(n,{s_i}, {t_i}), we have

$$\frac{\partial^2 TP}{\partial t_i^2} = [(p(t_i)+ c_i - c_v(t_i)\theta\,(0) - [c_h - \theta\, c_v(t_i) - c_b]\beta(t_i - t)$$

$$+\int_{s_{i-1}}^{t_i} \{[p''(t_i) - c_v''] \beta(t_i - t) + [p(t_i) - c_v(t_i) - c_h]\beta(t_i - t)\}$$

$$+[p(t_i)+ c_i - c_v(t_i) - c_v(t_i)]\ \beta(t_i - t)\ f(t)\ dt$$

$$=\int_{t_i}^{\infty} \{p''(t_i) + [\theta c_v'(ti) - c_v''(t_i) - \theta\,(c_h + \theta\, c_v(t_i) - c_v'(t_i)]e^{\theta(s-t_i)}\} f(t)\ dt$$

$$\frac{\partial^2 TP}{\partial s_{i-1}\,\partial t_i} = -P_{st}(s_{i-1}, t_i)\ f(s_{i-1}) > 0$$

$$\frac{\partial^2 TP}{\partial t_i\,\partial s_i} = R_{st}\,(t_i, s_i)\ f(s_i) > 0$$

$$\frac{\partial^2 TP}{\partial s_i^2} = \{[p(t_{i+1})+ c_i - c_v(t_{i+1}) - c_b\,(t_{i+1} - s_i)\theta(t_{i+1} - s_i)\text{-}\ c_h\,\beta(t_{i+1} - s_i)$$

$$[c_h + \theta\, c_v(t_i)]e^{\theta(s-t_i)}\} f(s_i) \qquad \text{(C.1)}$$

Let Δk be the principal minor of order k, then, under condition (4.7)- (4.8), it is clear that

$$\Delta_1 = \frac{\partial^2 TP}{\partial t_1^2} \le -\frac{\partial^2 TP}{\partial s_1\,\partial t_1} = -Rt_1(t_1,s_1)\ f\,(s_1) < 0, \qquad \text{(C.2)}$$

Which implies that $\Delta_1+(\partial^2 TP/\partial s_1\partial t_1)<0$:

$$\Delta_2 = \frac{\partial^2 TP}{\partial t_1^2}\frac{\partial^2 TP}{\partial s_1^2} - \frac{\partial^2 TP}{\partial s_1\,\partial t_1}\frac{\partial^2 TP}{\partial t_1\,\partial s_1} \ge -\frac{\partial^2 TP}{\partial s_2\,\partial s_1}\Delta_1 - \frac{\partial^2 TP}{\partial t_1\,\partial s_1}\left(\Delta_1 + \frac{\partial^2 TP}{\partial t_1\,\partial s_1}\right) > 0, \qquad \text{(C.3)}$$

Which implies that $\Delta_2 + (\partial^2 TP/\partial t_2 \partial s_1)\Delta_1 > 0$. For principal minor of higher order, i 2, 3, . . ., it is not difficult to show that they satisfy the following recursive relation:

$$\Delta_{2i-1} = \frac{\partial^2 TP}{\partial t_i^2}\Delta_{2i-2} - \left[\frac{\partial^2 TP}{\partial t_i \, \partial s_{i-1}}\right]^2 \Delta_{2i-3},$$

$$\Delta_{2i} = \frac{\partial^2 TP}{\partial s_i^2}\Delta_{2i-1} - \left[\frac{\partial^2 TP}{\partial s_i \, \partial t_i}\right]^2 \qquad \text{(C.4)}$$

With the initial $\Delta_0 = 1$. From (4.7) - (4.8) and the relation between second-order partial derivatives, we have

$$\Delta_{2i-1} \leq -\frac{\partial^2 TP}{\partial s_i \, \partial t_i}\Delta_{2i-1} - \frac{\partial^2 TP}{\partial t_i \, \partial s_{i-1}}\left(\Delta_{2i-2} + \frac{\partial^2 TP}{\partial t_i \, \partial s_{i-1}}\Delta_{2i-3}\right)$$

$$\Delta_{2i-1} \geq -\frac{\partial^2 TP}{\partial s_i \, \partial t_i}\Delta_{2i-1} - \frac{\partial^2 TP}{\partial t_i \, \partial s_{i-1}}\left(\Delta_{2i-2} + \frac{\partial^2 TP}{\partial t_i \, \partial s_{i-1}}\Delta_{2i-3}\right)$$

For i = 2 in (C.5), we obtain

$$\Delta_3 + \frac{\partial^2 TP}{\partial s_2 \, \partial t_2}\Delta_2 \leq -\frac{\partial^2 TP}{\partial t_2 \, \partial s_1}\left(\Delta_2 + \frac{\partial^2 TP}{\partial t_2 \, \partial s_{i-1}}\Delta_1\right) < 0,$$

$$\Delta_4 + \frac{\partial^2 TP}{\partial t_2 \, \partial s_2}\Delta_3 \leq -\frac{\partial^2 TP}{\partial s_2 \, \partial t_2}\left(\Delta_3 + \frac{\partial^2 TP}{\partial s_2 \, \partial t_2}\Delta_2\right) > 0, \qquad \text{(C.6)}$$

Thus,

$$\Delta_3 + \frac{\partial^2 TP}{\partial s_2 \, \partial t_2}\Delta_2 < 0,\ \Delta_4 > \frac{\partial^2 TP}{\partial t_2 \, \partial s_2}\Delta_3 > 0 \qquad \text{(C.7)}$$

Proceeding inductively, we have

$$\Delta_{2i-1} + \frac{\partial^2 TP}{\partial s_i \, \partial t_i}\Delta_{2i-2} < 0,\quad \Delta_{2i} + \frac{\partial^2 TP}{\partial t_{i+1} \, \partial s_i}\Delta_{2i-2} > 0, \qquad \text{(C.8)}$$

Therefore, $\Delta_{2i-1} < 0$ and $\Delta_{2i} > 0$, for i = 2, 3, ... This completes the proof.

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